

Search for the variation of the
fundamental constants in atomic &
molecular experiments
(Slow drifts & fast oscillations)

M G Kozlov

Ameland
June 2019

Energy of atoms and molecules

Atomic unit of energy, Hartree:

$$E_H = \frac{m_e e^4}{(4\pi\epsilon_0 \hbar)^2} = \frac{m_e e^4}{(2\epsilon_0 h)^2} \approx 27 \text{ eV}$$

Electronic energy of an atom, or a molecule:

$$E_{\text{el}} = E_H [C_{e,0} + C_{e,1}(\alpha Z)^2 + \dots]$$

Vibrational and rotational energy of a molecule

$$E_{\text{vib}} = C_v E_H \mu^{1/2}, \quad E_{\text{rot}} = C_r E_H \mu$$

$$C_v = C_{v,0} + C_{v,1}(\alpha Z)^2 + \dots$$

All these energies depend on one dimensionful constant E_H and two dimensionless constants, the fine structure constant and electron to proton mass ratio:

$$\alpha = \frac{e^2}{2\epsilon_0 hc} \quad \mu = \frac{m_e}{m_p}$$

Transition frequencies of atoms and molecules

As long as all energies and frequencies are proportional to E_H , then the ratios of any atomic and molecular frequencies are independent of E_H and depend only on the dimensionless constants α and μ .

Frequency of optical cavity

The length of the cavity L is proportional to the lattice constant a and, therefore, to the Bohr radius r_0 :

$$r_0 = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2}$$

Resonant frequency of the cavity :

$$\nu_{\text{cav}} = C_c \frac{c}{r_0} = C_c \frac{m_e e^2 c}{4\pi\epsilon_0 \hbar^2} = C_c \frac{E_H}{\hbar\alpha}$$

Relativistic corrections:

$$C_c = C_{c,0} + C_{c,1}(\alpha Z)^2 + \dots$$

The ratio of the atomic transition frequency to the cavity frequency is:

$$\frac{\delta(\nu_{\text{at}}/\nu_{\text{cav}})}{(\nu_{\text{at}}/\nu_{\text{cav}})} = \frac{\delta\alpha}{\alpha} + \mathcal{O}(\alpha Z)^2$$

This ratio also depends only on the dimensionless constant α .

Sensitivity coefficients to α -variation

Dimensional sensitivity coefficients q to α -variation were introduced by Dzuba, Flambaum & Webb in 1992:

$$\delta E = 2q_\alpha \frac{\delta\alpha}{\alpha_0} \quad \hbar\delta\omega_{i,f} = 2(q_\alpha^f - q_\alpha^i) \frac{\delta\alpha}{\alpha_0} \equiv 2q_\alpha^{i,f} \frac{\delta\alpha}{\alpha_0}$$

It is also convenient to define dimensionless coefficients K which give relative frequency change:

$$\frac{\delta\omega}{\omega_0} = K_\alpha \frac{\delta\alpha}{\alpha_0}, \quad K_\alpha = \frac{2q_\alpha}{\omega_0} \quad K_\alpha = \frac{\partial\omega}{\partial\alpha} \frac{\alpha}{\omega}$$

Sensitivity coefficients to μ -variation

By analogy we can define sensitivity coefficients to μ -variation :

$$\delta E = q_{\mu} \frac{\delta \mu}{\mu_0}, \quad \frac{\delta \omega}{\omega_0} = K_{\mu} \frac{\delta \mu}{\mu_0}, \quad K_{\mu} = \frac{q_{\mu}}{\omega_0}$$

Dependency on the units

The dimensional factors q obviously depend on the choice of units. It is sometimes assumed that K factors do not depend on the units. This is not true. Let us compare, for example, relativistic units ($c=1$) and atomic units ($e=1$):

$$\tilde{\omega} = \alpha^2 \omega \quad \tilde{K}_\alpha = K_\alpha + 2$$

However, if we take a ratio of two frequencies, then its dependence on the constants does not depend on the choice of units:

$$\frac{\delta (\omega_m / \omega_n)}{\omega_m / \omega_n} = (K_\alpha^m - K_\alpha^n) \frac{\delta \alpha}{\alpha}$$

Sensitivity coefficients to α -variation for an optical cavity

The length of the cavity L_c is linked to the lattice constant a :

$$\frac{\delta L_c}{L_c} = \frac{\delta a}{a} = T_\alpha^a \frac{\delta \alpha}{\alpha}$$

Then, we can write for the resonant frequency of the cavity in atomic units (where $c=\alpha^{-1}$):

$$\frac{d\omega_c^N}{d\alpha} = \frac{d}{d\alpha} \left(\frac{\pi N}{\alpha L_c} \right) = -\omega_c^N \left(\frac{1}{\alpha} + \frac{1}{L_c} \frac{dL_c}{d\alpha} \right) \Rightarrow K_\alpha^c = - (1 + T_\alpha^a)$$

Assuming that relativistic corrections for the lattice are small, we get for the ratio of atomic frequency and the frequency of the cavity:

$$\frac{\delta (\omega_{\text{at}}/\omega_c^N)}{\omega_{\text{at}}/\omega_c^N} \approx (K_\alpha^{\text{at}} + 1) \frac{\delta \alpha}{\alpha}$$

Up to now we implicitly assumed that “fundamental constants” may only change very slowly at the times much longer than the response time of the systems, whose spectra we are studying.

Fast oscillations of “constants”

Atom is averaging values of the constants during the lifetime of atomic levels:

$$\tau_{\text{at}} = \Gamma^{-1} \quad \nu_{\text{at}} = C_{\text{at}} \left\langle \frac{E_H}{h} \right\rangle_{\tau_{\text{at}}}$$

For the cavity there are two characteristic response times

$$\tau_{\text{cav},1} \propto QL/c \quad \tau_{\text{cav},2} \approx L/v_s$$

If the quality factor $Q < c/v_s$, then $\tau_{\text{cav}} = \tau_{\text{cav},2} > \tau_{\text{cav},1}$.

The ration of the atomic transition frequency to the cavity frequency is:

$$\nu_{\text{cav}} = C_c \left\langle \frac{E_H}{h\alpha} \right\rangle_{\tau_{\text{cav}}}$$

Resent experiment with Cs and optical cavity

[D. Antypas et al, arXiv:1905.02968]

6s – 6p_{3/2} transition in Cs is compared to the resonant frequency of the 12 cm stainless steel cavity.

$$\tau_{\text{at}} = 30.5 \cdot 10^{-9} \text{ s.} \quad \tau_{\text{cav}} = 2 \cdot 10^{-5} \text{ s.}$$

If the constant oscillate at the frequency ν_s , then result of the comparison depends on this frequency

$$\nu_s \ll \tau_{\text{cav}}^{-1}$$

$$\frac{\delta(\nu_{\text{at}}/\nu_{\text{cav}})}{(\nu_{\text{at}}/\nu_{\text{cav}})} = \frac{\delta\alpha}{\alpha}$$

$$\tau_{\text{cav}}^{-1} \ll \nu_s \ll \tau_{\text{at}}^{-1}$$

$$\frac{\delta(\nu_{\text{at}}/\nu_{\text{cav}})}{(\nu_{\text{at}}/\nu_{\text{cav}})} = \frac{\delta E_H}{E_H}$$

$$\tau_{\text{at}}^{-1} \ll \nu_s$$

$$\frac{\delta(\nu_{\text{at}}/\nu_{\text{cav}})}{(\nu_{\text{at}}/\nu_{\text{cav}})} = \text{const}$$

We see that for intermediate frequencies of oscillations of FC the measured frequency ratio depends on the variation of the dimensionful parameter E_H .

The atom feels instantaneous value the parameter E_H , while the cavity remembers the average value. Variation is then defined as:

$$\delta E_H = E_H - \langle E_H \rangle$$

Example: DM model with light scalar field

SM Lagrangian for photon and electron fields:

$$\mathcal{L}_{\text{SM}} = -\frac{e^2 c}{8h\alpha} F_{\mu\nu} F^{\mu\nu} + i\hbar c \bar{\psi}_e \gamma^\mu \partial_\mu \psi_e - m_e c^2 \bar{\psi}_e \psi_e + ec \bar{\psi}_e \gamma^\mu A_\mu \psi_e$$

Model interaction with a light scalar field:

$$\mathcal{L}_{\text{int}} = \varphi \left[d_e \frac{e^2 c}{8h\alpha} F_{\mu\nu} F^{\mu\nu} - d_{m_e} m_e c^2 \bar{\psi}_e \psi_e \right]$$

This interaction modifies I & III terms of SM Lagrangian. This modification can be absorbed in two redefined field-dependent FC constants:

$$\begin{aligned} \alpha(\varphi) &= \alpha(1 + d_e \varphi), \\ m_e(\varphi) &= m_e(1 + d_{m_e} \varphi) \end{aligned}$$

Fast oscillations induced by the field φ

The field φ modifies α and m_e , so we need to express E_H in terms of these parameters:

$$E_H = m_e c^2 \alpha^2$$

Then variation of E_H is given by the expression:

$$\frac{\delta E_H}{E_H} \left(= \frac{\delta m_e}{m_e} + 2 \frac{\delta \alpha}{\alpha} \right) = \varphi (d_{m_e} + 2d_e)$$

Conclusions

- If we allow fundamental constants to change slowly, then any spectroscopic experiment with atoms and molecules are sensitive to variations of dimensionless constants, primarily, α and μ .
- When we study fast oscillations of the frequency ratios, we may be sensitive to the variation of dimensionful constant E_H . Interpretation of such experiments in terms of variation of FC is not fully justified and model dependent. *Therefore, it is better to refer to the couplings in particular models.*

Thank you!