

Recent advances in relativistic coupled cluster methods for open shell states

A. V. Oleynichenko

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Current challenges in chemical physics and theoretical chemistry – 2014
5th July, 2024

Why?

- ▶ chemistry and spectroscopy of **actinides and superheavy** elements
- ▶ **active laser media**; sources of light; chromophores, luminophores
- ▶ searches for \mathcal{P}, \mathcal{T} -odd fundamental interactions (physics beyond the Standard model)
- ▶ thermodynamics, physical and chemical **properties of actinide compounds**
- ▶ fine structure effects in spectra of light elements; spin-forbidden transitions
- ▶ the Periodic table for the most heavy chemical elements
- ▶ optical and magnetic properties of f -element compounds
- ▶ laser cooling and assembly of **cold molecules**
- ▶ ...

a clear understanding of the experiment is impossible without a theoretical model!

but: models for d - and f -elements have to be very complicated...

Single-reference vs multi-reference approaches

$$\hat{H}_e |\psi_n\rangle = E_n |\psi_n\rangle$$

- ▶ **single-reference (SR):** one leading Slater determinant
+ determinants excited wrt it

$$\psi_n = \Phi_0 + \sum_K C_K \Phi_K$$

- ▶ **multi-reference (MR):** several leading configurations (determinants)
+ determinants excited wrt them

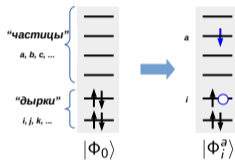
$$\psi_n = \sum_{\mu} C_{\mu} \Phi_{\mu} + \sum_K C_K \Phi_K$$

Single reference coupled cluster method

- ▶ exponential Ansatz for an electronic wavefunction:

$$|\psi\rangle = e^T |\Phi_0\rangle$$

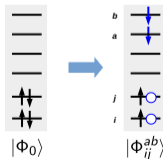
- ▶ single excitations:



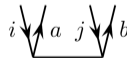
$$T_1 = \sum_{ia} t_i^a \{a_a^\dagger a_i\}$$



- ▶ double excitations:



$$T_2 = \frac{1}{4} \sum_{ijab} t_{ij}^{ab} \{a_a^\dagger a_i a_b^\dagger a_j\}$$



objective: calculate amplitudes $t_i^a, t_{ij}^{ab}, \dots$

Single reference coupled cluster method

Energy expression and amplitude equations

- ▶ wave function:

$$|\psi\rangle = e^T |\Phi_0\rangle$$

- ▶ Schrödinger equation \Rightarrow Bloch equation:

$$H|\psi\rangle = E|\psi\rangle \quad \Rightarrow \quad (He^T)_c |\Phi_0\rangle = E_{corr} |\Phi_0\rangle$$

$(He^T)_c$ = only connected Brando diagrams

$$E = E_{HF} + E_{corr}$$

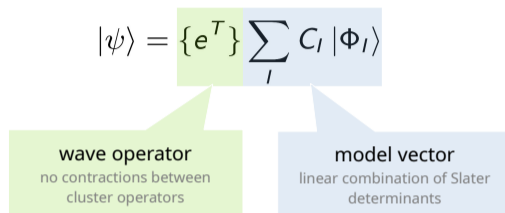
- ▶ projection onto $\langle\Phi_0| \Rightarrow$ expression for the correlation energy:

$$E_{corr} = \langle\Phi_0|(He^T)_c|\Phi_0\rangle$$

- ▶ projection onto $\langle\Phi_i^a|, \langle\Phi_{ij}^{ab}| \Rightarrow$ amplitude equations (CCSD):

$$\begin{cases} \langle\Phi_i^a|(He^T)_c|\Phi_0\rangle = 0 & \text{singles (S)} \\ \langle\Phi_{ij}^{ab}|(He^T)_c|\Phi_0\rangle = 0 & \text{doubles (D)} \end{cases}$$

Fock-space multireference coupled cluster method



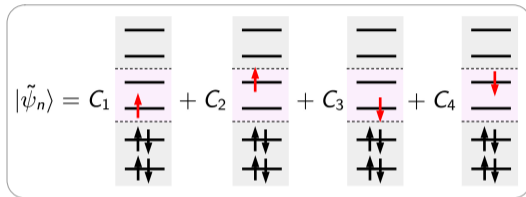
- ▶ wave operator $\Omega = \{e^T\} \Rightarrow$ dynamic correlation
- ▶ model vector $|\tilde{\psi}\rangle = \sum_{\mu} C_{\mu} |\Phi_{\mu}\rangle \Rightarrow$ multiconfigurational nature of a state
- ▶ effective Hamiltonian \tilde{H} acts within a model space:

$$H |\psi_n\rangle = E_n |\psi_n\rangle \quad \Rightarrow \quad \tilde{H} |\tilde{\psi}_n\rangle = E_n |\tilde{\psi}_n\rangle$$

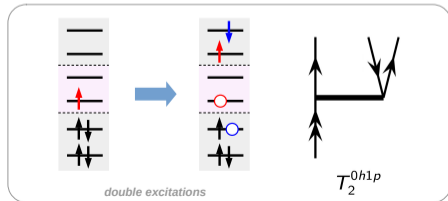
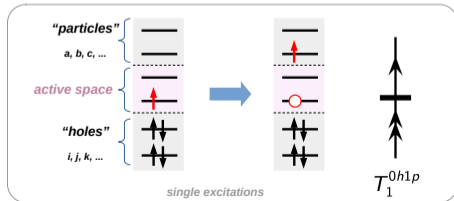
Fock-space multireference coupled cluster method

The $0h1p$ sector: one particle above the closed shell (CCSD model)

- ▶ model vector in the $0h1p$ sector:



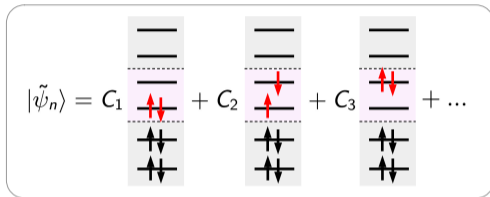
- ▶ cluster operator $T = T^{0h0p} + T^{0h1p}$ (CCSD model):



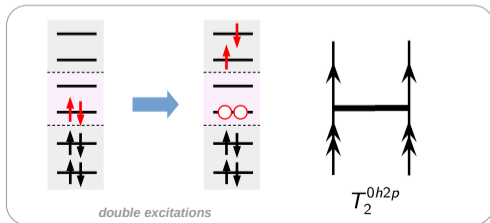
Fock-space multireference coupled cluster method

The $0h2p$ sector: two particles above the closed shell (CCSD model)

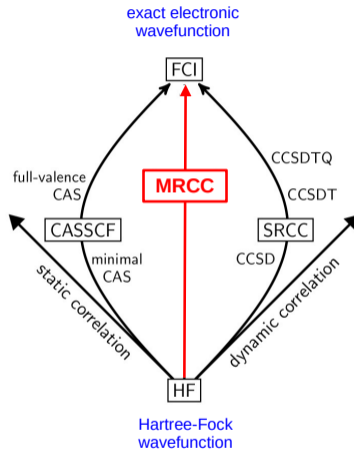
- ▶ model vector in the $0h2p$ sector:



- ▶ cluster operator $T = T^{0h0p} + T^{0h1p} + T^{0h2p}$ (CCSD model):



single-reference vs multi-reference



- ▶ multi-reference CC: well-balanced treatment of correlation effects of different nature

Hamiltonian: (generalized) relativistic pseudopotentials (GRPP)

- ▶ valence and outercore electrons are described by the two-component effective Hamiltonian:

$$\hat{H}^{RPP} = \sum_i \left(-\frac{\Delta_i}{2} + \sum_{\alpha} \left(-\frac{z_{\alpha}}{|\mathbf{R}_{\alpha} - \mathbf{r}_i|} + \hat{U}_{\alpha}(i) \right) \right) + \sum_{i>j} \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|}$$

i, j – sum over electrons

α – sum over nuclei

z_{α} – effective charge of an inner core of an atom α , $z_{\alpha} = Z_{\alpha} - N_{\text{inner-core elec-s}}$

- ▶ pseudopotential \hat{U} substitute inner core electrons and can effectively include:
 - ▶ scalar-relativistic effects (\sim mass-velocity + Darwin)
 - ▶ spin-orbit interaction
 - ▶ Breit e-e interaction
 - ▶ finite nuclear charge distribution (Fermi model)
 - ▶ leading QED effects (model operator = self-energy + vacuum polarization)
- ▶ the most exact formulation: generalized relativistic pseudopotential (typical error of EEs $< 50 \text{ cm}^{-1}$)
- ▶ available in DIRAC and CP2K via the LIBGRPP library!

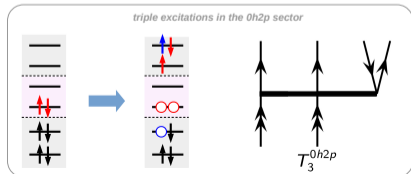
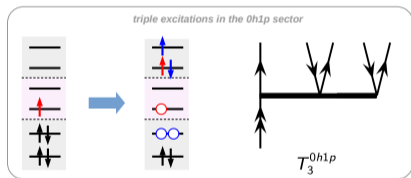
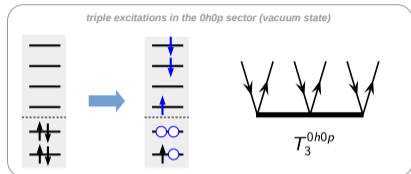
Challenges – 2017

- ▶ accounting for triples and three open shells
- ▶ transition moments and other off-diagonal properties
- ▶ density matrices
- ▶ impurity centers in crystals

also:

- ▶ intruder state problem \Rightarrow amplitude equations typically fail to converge (A. V. Zaitsevskii)

Accounting for triples: the FS CCSDT model



- ▶ cluster operator:

$$T = T_1 + T_2 + T_3$$

- ▶ number of floating-point operations – $O(N^8)$

N one-particle basis functions (spin-orbitals or spinors)

- ▶ correlation of “added” electrons in the $0h1p$ and $0h2p$ sectors

Pilot applications of the FS CCSDT model

Ionization potentials and excitation energies of the Tl and Pb atoms, cm^{-1}

A. Oleynichenko, A. Zaitsevskii, L. V. Skripnikov, E. Eliav, *Symmetry*, 12(7), 1101 (2020)

Table 1. Deviations of the calculated ionization potentials (IP) and excitation energies (EE) of neutral thallium and lead and lead cation (cm^{-1}) from the experimental values. FS-RCCSD/LB+T/SB stands for the combined scheme (8).

State		Exptl [84]	IH-FS- RCCSD [47]	FS- RCCSD/LB	SDT-1	FS-RCCSD/LB + T/SB			SDT
						SDT-1'	SDT-2	SDT-3	
Tl, ground state $6s^2 6p^2 \ ^2P_{1/2}$									
IP		49,266		-56	-38	-38	-204	-151	-32
EE	$6s^2 6p^2 \ ^2P_{3/2}$	7793		-112	23	23	1	9	-31
Pb ⁺ , ground state $6s^2 6p^2 \ ^2P_{1/2}$									
IP		121,245	-168	-143	-28	-28	-190	-158	-59
EE	$6s^2 6p^2 \ ^2P_{3/2}$	14,081	-196	-136	25	25	12	14	-42
Pb, ground state $6s^2 6p^2 \ ^3P_0$									
IP		59,819	-543	364	-4	-4	-37	-336	7
EE	$6s^2 6p^2 \ ^3P_1$	7819	-288	-302	76	5	-4	-3	-28
	3P_2	10,650	-343	-235	1	1	1	1	13
	1D_2	21,458	-605	-394	215	158	158	167	5
	1S_0	29,467	-208	414	170	248	291	302	173

CCSD → CCSDT
few meV accuracy achieved

the way towards the most precise atomic calculations ever?

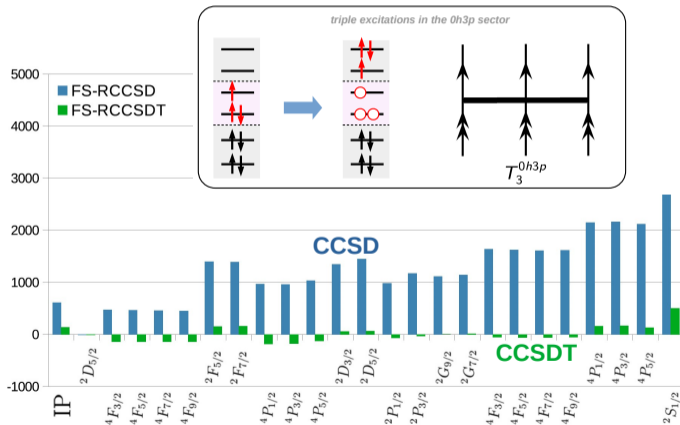
A. Landau, E. Eliav, Y. Ishikawa, U. Kaldor, *J. Chem. Phys.* 114, 2977 (2001)

J. E. Sansonetti, W. C. Martin, *J. Phys. Chem. Ref. Data*, 34, 1559 (2005)

Pilot applications of the FS CCSDT model

Lanthanum atom energy levels, configurations $6s^25d$, $6s^15d^2$, $5d^3$

Deviations of calculated excitation energies from experiment, cm^{-1}



$0h3p$ sector: refinement of the quadrupole moments of Bi isotopes

L. V. Skripnikov, A. V. Oleynichenko, A. V. Zaitsevskii, D. E. Maison, A. E. Barzakh, *Phys. Rev. C* 104(3), 034316 (2021)

- ▶ electromagnetic moments of heavy nuclei are known with a **large uncertainty (up to 20%)!**
- ▶ **how to extract nuclear electric quadrupole moment Q** from experimentally measured hyperfine constant B :

$$Q \text{ [b]} = \frac{B \text{ [MHz]}}{234.9648867 \cdot q \text{ [a.u.]}}$$

B – electric quadrupole hyperfine-structure constant

q – electric field gradient on a nucleus

- ▶ $6p^3$ **electronic states** of the Bi atom are **multiconfigurational** (the $0h3p$ sector of the Fock space)

$0h3p$ sector: refinement of the quadrupole moments of Bi isotopes

L. V. Skripnikov, A. V. Oleynichenko, A. V. Zaitsevskii, D. E. Maison, A. E. Barzakh, *Phys. Rev. C* 104(3), 034316 (2021)

- ▶ how to extract nuclear electric quadrupole moment Q from experimentally measured hyperfine constant B :

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TABLE I. The calculated values of the electric field gradient in a.u.(= E_H/a_B^2) for the ground $4S_{3/2}^o$ and excited $2P_{3/2}^o$ electronic states of neutral bismuth and the deduced values of the NQM of ^{209}Bi .

	$6p^3 \ 4S_{3/2}^o$	$6p^3 \ 2P_{3/2}^o$
	EFG:	
FS-CCSD	2.983	-10.292
basis set correction	0.055	-0.050
FS-CCSDT – FS-CCSD	0.117	0.276
Breit contribution	-0.058	0.088
Total	3.097	-9.978
B , MHz [36]	-305.067(2)	978.638(10)
$Q(^{209}\text{Bi})$, mb	-419	-417

Challenges – 2017

- ▶ accounting for triples and three open shells
- ▶ transition moments and other off-diagonal properties
- ▶ density matrices
- ▶ impurity centers in crystals

Effective operator of a property

A. V. Oleynichenko, A. V. Zaitsevskii, S. V. Kondratyev, E. Eliav, *Opt. Spectrosc.*, 131(11), 1471 (2023)

- ▶ basic idea: use truncated expression for the CC wavefunction:

$$|\psi_n\rangle = \{e^T\} |\tilde{\psi}_n\rangle \approx \left(1 + T + \frac{\{T^2\}}{2}\right) |\tilde{\psi}_n\rangle$$

- ▶ effective property operator \tilde{O} in the 2nd order in T :

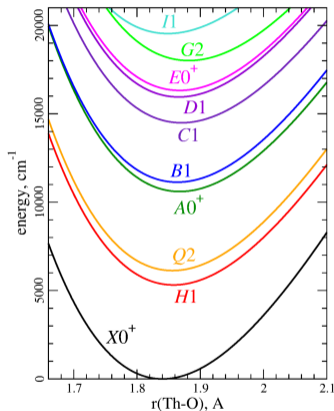
$$\tilde{O} \approx \left(O + T^\dagger O + OT + \frac{\{(T^\dagger)^2\}}{2} O + T^\dagger OT + O \frac{\{T^2\}}{2} - (T^\dagger T)_{cl} O\right)_{cl, conn}$$

- ▶ exact mutual cancellation of all disconnected diagrams
- ▶ intensities of absorption and emission $\sim |\langle \psi_n | \hat{d} | \psi_m \rangle|^2$
- ▶ one calculation \Rightarrow matrix elements for all pairs of electronic states

See also: A. Zaitsevskii, A. V. Oleynichenko, E. Eliav. *Mol. Phys.* e2236246 (2023)

Application: excited states of the ThO molecule

Term energies T_e , cm^{-1} [A. Zaitsevskii, A. V. Oleynichenko, E. Eliav, *Mol. Phys.* e2236246 (2023)]



	exptl, cm^{-1}	deviation from experiment, cm^{-1}		
		RPP/FS-CCSD	DC/FS-CCSD	
		0h2p	0h2p	1h1p
H(i)1	5317	104	700	-149
Q(i)2	6128	97	738	-62
A(ii)0 ⁺	10601	242	691	1098
B(ii)1	11129	302	927	-
C(iii)1	14490	424	1698	-39
D(iv)1	15946	440	1698	-
E(iii)0 ⁺	16320	312	960	-1950
G(iv)2	18010	165	-	-
F(iv)0 ⁺	18338	431	-	-
I(vi)1	19539	367	-	-

[present work]

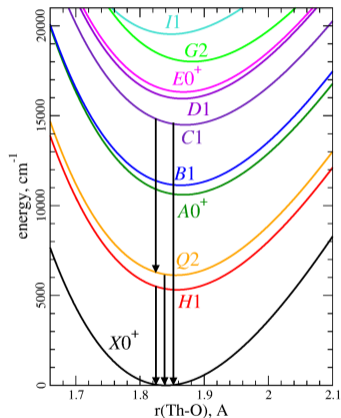
[Tecmer et al, *PCCP*, 2018]

Why ThO? One of the objects to search for electron electric dipole moment

ACME Collaboration, *Nature*, 562, 355 (2018)

Application: excited states of the ThO molecule

Lifetimes of excited states [A. Zaitsevskii, A. V. Oleynichenko, E. Eliav, *Mol. Phys.* e2236246 (2023)]



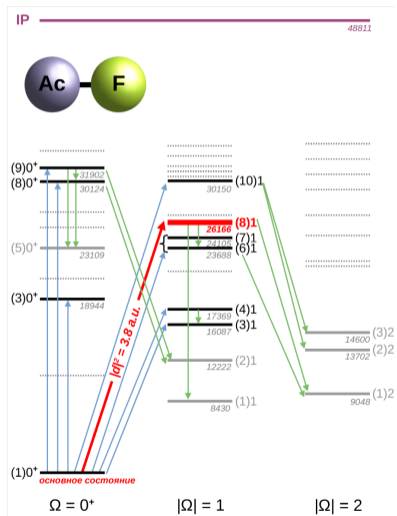
	exptl	theory
$H \rightarrow X$	$4.2 \pm 0.5 \text{ ms}^a$	3.82 ms
$Q \rightarrow X$	$> 62 \text{ ms}^b$	177 ms
$C \rightarrow \dots$	$> 480 \text{ ns}^c$	400 ns
$C \rightarrow Q$	$468 \pm 30 \text{ ns}^d$	5.49 μs

^a D. G. Ang et al, *Phys. Rev. A* 106, 022808 (2022); ^b X. Wu et al, *New J. Phys.* 22, 023013 (2020)

^c N. R. Hutzler et al, *Phys. Chem. Chem. Phys.* 13, 18976 (2011); ^d D. L. Kokkin et al, *Phys. Rev. A* 91, 042508 (2015)

Spectroscopy of the AcF molecule

L. V. Skripnikov et al, *J. Chem. Phys.* 159, 124301 (2023)



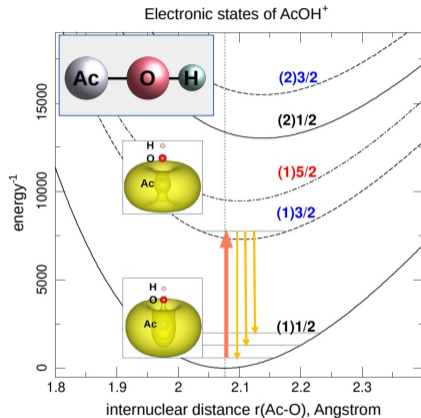
- ▶ promising object for the searches of the \mathcal{T}, \mathcal{P} -odd nuclear Schiff moment (^{225}Ac , ^{227}Ac)

L. V. Skripnikov et al, *PCCP* 22, 18374 (2020)

- ▶ low-lying states: two electrons above the closed-shell vacuum AcF^{2+}
- ▶ ~ 80 states below 43000 cm^{-1}
- ▶ the most intense transitions were predicted $\max |\langle \psi_i | \mathbf{d} | \psi_f \rangle|^2$
- ▶ the (8)1 state was studied experimentally at CRIS/ISOLDE (CERN)

the first experimental study of the AcF molecule was not possible without theoretical modeling!

AcOH⁺: first prediction of a laser-coolable polyatomic ion



a promising system for the new generation of experiments to search \mathcal{T} , \mathcal{P} -violation effects
⇒ physics beyond the Standard Model

A. V. Oleynichenko, L. V. Skripnikov, A. V. Zaitsevskii, V. V. Flambaum *Phys. Rev. A*, 105, 022825 (2022).

Challenges – 2017

- ▶ accounting for triples and three open shells
- ▶ transition moments and other off-diagonal properties
- ▶ density matrices
- ▶ impurity centers in crystals

Approximate method to calculate FS CC density matrices

- ▶ exact expression:

$$(\gamma_N)_{qp} = \frac{N_i}{N_f} \langle \tilde{\psi}_i^{\perp\perp} | (\Omega^\dagger \Omega)^{-1} \Omega^\dagger \{a_p^\dagger a_q\} \Omega | \tilde{\psi}_f \rangle \quad N_i = \langle \tilde{\psi}_i | \Omega^\dagger \Omega | \tilde{\psi}_f \rangle^{1/2}$$

$$\Omega = \{e^T\} = \left\{1 + T + \frac{TT}{2} + \frac{TTT}{6} + \dots\right\}$$

- ▶ use the Taylor expansion to express the inverse metric operator:
only for the Fock space sectors $0hPp$ and $Hh0p$!

$$(\Omega^\dagger \Omega)^{-1} \approx 1 - \Omega^\dagger \Omega + \dots$$

- ▶ we retain only terms linear and quadratic in T :

$$(\Omega^\dagger \Omega)^{-1} \Omega^\dagger \{a_p^\dagger a_q\} \Omega \approx \boxed{\Omega^\dagger \{a_p^\dagger a_q\} \Omega - (T^\dagger T)_{cl} \{a_p^\dagger a_q\}}$$

- ▶ exact mutual cancellation of disconnected diagrams
- ▶ very low computational cost!

How to use density matrices and natural spinors?

not density matrices themselves, but natural [transition] spinors are of primary interest

- ▶ contracted relativistic ANO-type basis sets for excited state calculations
 - ⇒ before: scalar-relativistic CCSD/CCSD(T) density matrices to construct ANOs (CFOUR)
 - ⇒ now: fully relativistic CCSD DMs (DIRAC+EXP-T)
 - ⇒ future: basis sets with nearly the same contraction error for all target electronic states?
- ▶ natural spinors as a basis for FS CCSDT calculations?
- ▶ visualization of excited states and electronic transitions?

Challenges – 2017

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- ▶ transition moments and other off-diagonal properties
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- ▶ impurity centers in crystals

Localized excitations on f -element ions Ce^{3+} , Th^{3+} in xenotime YPO_4 crystals (Y. V. Lomachuk)

- ▶ natural xenotime contains Th and U impurities
- ▶ radiation resistant, no metamictization
- ▶ very wide bandgap (> 8.6 eV)

- ▶ YPO_4 doped with lanthanide ions:
 - ▶ laser active media, scintillators, luminophores ...
 - ▶ large amount of experimental data:
 $YPO_4:Ce^{3+}$, $YPO_4:Pr^{3+}$, $YPO_4:Nd^{3+}$, $YPO_4:Yb^{3+}$, ...
 - ▶ energy and charge transfer processes between lanthanide sites

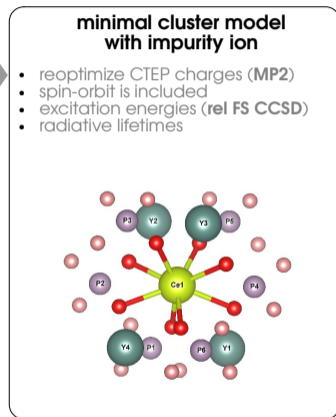
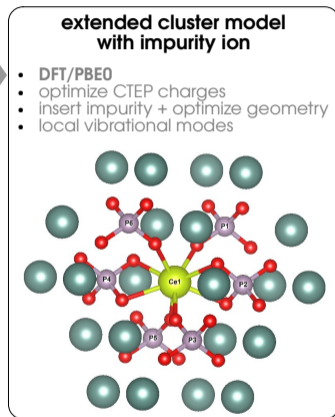
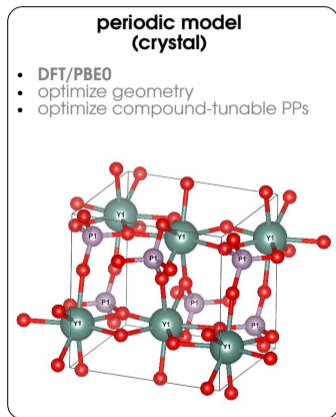
- ▶ YPO_4 doped with actinide ions:
 - ▶ immobilization of highly radioactive waste
 - ▶ nuclear clock on the isomeric transition in ^{229}Th
M. G. Kozlov et al, *Phys. Rev. A* 109, 042806 (2024)



Xenotime crystal

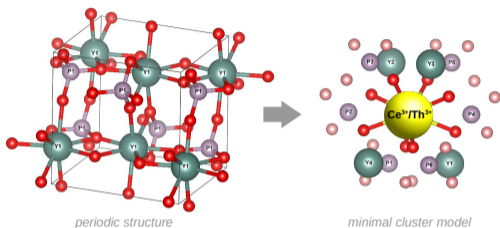
Locality: Novo Horizonte, Brazil

Minimal cluster model of an impurity center

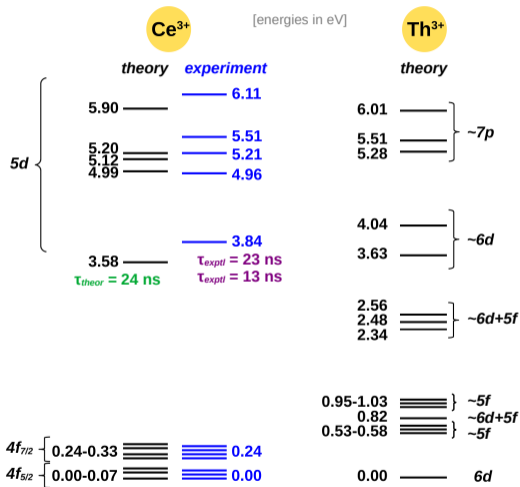


CTEP = Compound-Tunable Effective Potential

Excitation energies and radiative lifetimes of excited states in crystal

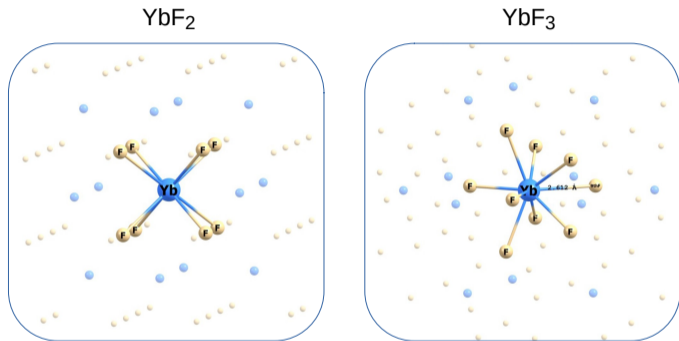


- ▶ errors of order 0.2 – 0.3 eV
- ▶ ground state of Th^{3+} in crystal – $6d^1$
- ▶ minimal cluster model calculations: FS RCCSD
- ▶ correction for the cluster model size: TD-DFT ☹️
- ▶ the interplay of the crystal field and spin-orbit interaction



Coming soon: relativistic CC to calculate X-ray emission shifts in solids

(P. A. Khadeeva, V. M. Shakhova, A. V. Titov)



- ▶ X-ray emission shift: direct probe of the state of an Atom-in-Compound (AiC)
- ▶ compound-tunable embedding potential
- ▶ configurations f^{14} (Yb²⁺) and f^{13} (Yb³⁺) \Rightarrow Kramers-unrestricted CCSD(T) works well

Fig. from: V. M. Shakhova et al, *Phys. Chem. Chem. Phys.* 24, 19333 (2022)

The EXP-T program package

The new program package EXP-T for coupled cluster calculations was developed at NRC “Kurchatov Institute” – PNPI

- ▶ electronic structure of atoms, molecules and defects in crystals
- ▶ written from scratch in C
- ▶ molecular integrals are imported from the DIRAC package
relativistic Hamiltonians: Schrödinger, Dirac-Coulomb(-Gaunt) DC(G), (generalized) pseudopotentials
- ▶ parallelization: OpenMP
- ▶ fast and flexible implementation of new models

The EXP-T program package

Implemented models and other features:

- ▶ Kramers-unrestricted relativistic coupled cluster theory
- ▶ open shells: Fock-space multireference coupled cluster
⇒ up to $3h0p$, $2h1p$, $1h2p$, $0h3p$
- ▶ CCSD, CCSD(T), **CCSDT-1,2,3**, **CCSDT** models
- ▶ analytic density matrices for single-reference relativistic CCSD and **CCSD(T)**
- ▶ **approximate [transition] density matrices for FS CCSD**
- ▶ **property calculations**
⇒ transition moments: lifetimes and intensities in spectra
⇒ hyperfine structure

* unique features

The EXP-T program package

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About

The EXP-T program package is designed for high-precision modeling of molecular electronic structure using the relativistic Fock space multireference coupled cluster method (FS-RCC). EXP-T is written from scratch in the C99 programming language and is currently focused on Unix-like systems.

Readme LGPL-2.1 license

10 stars

2 watching

1 fork

File	Description	Last Commit
docs	direct calculation of properties in the 0h1p and 0h2p sectors	5 months ago
examples	direct calculation of properties in the 0h1p and 0h2p sectors	5 months ago
openblas	testing with ctest + refactoring of CC iterative solution in all sectors	6 months ago
scripts	expt_spectrum.py script	2 months ago
src	expt_spectrum.py script	2 months ago
test	expt_spectrum.py script	2 months ago
CMakeLists.txt	expt_spectrum.py script	2 months ago
LICENSE	Create LICENSE	2 weeks ago
README.md	Update README.md	3 years ago

README.md

The EXP-T program system

The EXP-T program package is designed for high-precision modeling of molecular electronic structure using the relativistic Fock space multireference coupled cluster method (FS-RCC). EXP-T is written from scratch in the C99 programming language and is currently focused on Unix-like systems.

Webpage of the EXP-T project:
<http://qchem.pnpi.spb.ru/expt>

Releases
No releases published

Packages
No packages published

Languages

- Fortran 48.2%
- Assembly 26.0%

<https://github.com/aoleynichenko/EXP-T>
<http://qchem.pnpi.spb.ru/expt>

Summary

breakthrough in accuracy = basis + CCSDT + GRPP + QED

but...

- ▶ max 3 unpaired electrons...
 - ⇒ multi-partitioning multireference PT (A. V. Zaitsevskii)
 - ⇒ other MRCC Ansätze
- ▶ extremely high computational cost
 - ⇒ localization techniques, DLPNO?
 - ⇒ tensor trains
- ▶ relaxed densities and nuclear gradients
- ▶ non-adiabatic couplings $\langle \psi_i | \nabla_{\mathbf{R}} | \psi_f \rangle$

Some other MRCC formulations

▶ Jeziorski-Monkhorst SU-MRCC

$$|\psi_n\rangle = \sum_{\mu} e^{T(\mu)} |\Phi_{\mu}\rangle \langle \Phi_{\mu}| \cdot |\tilde{\psi}_n\rangle$$

- ⇒ intruder-state problem
- ⇒ state-universal method (spectroscopy)
- ⇒ multi-vacuum = very costly ☹

▶ internally-contracted (ic) MRCC

$$|\psi_n\rangle = e^T \cdot |\tilde{\psi}_n\rangle$$

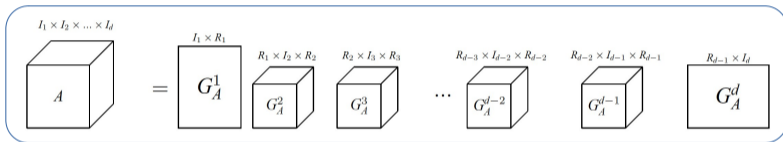
- ⇒ state-specific or state-selective
- ⇒ extremely complicated equations
- ⇒ automated generation of diagrams is needed
- ⇒ acceptable cost?
- ⇒ an open-source non-relativistic implementation exists (GeCCo)

Coming soon: tensor trains in coupled cluster theory (A. S. Romyantsev)

- ▶ all molecular integrals and amplitudes are stored as **multidimensional arrays** – “tensors”

$$\langle ij || ab \rangle \quad t_{ij}^{ab} \quad t_{ijk}^{abc}$$

- ▶ **tensor train decomposition** [I. V. Oseledets, *SIAM J. Sci. Comput.*, 33, 2295 (2011)]:
= matrix product state



$$A[i_1, i_2, \dots, i_d] = \sum_{\alpha_1, \dots, \alpha_{d-1}}^{R_1, \dots, R_{d-1}} \underbrace{G_1[1, i_1, \alpha_1]}_{TT\text{-core}} \times G_2[\alpha_1, i_2, \alpha_2] \times \dots \times G_d[\alpha_{d-1}, i_d, 1]$$

- ▶ **very efficient tensor contractions**
- ▶ still issues with the decomposition algorithm
- ▶ a new library was written in **Rust** from scratch (contractions, transpositions, sparse arithmetics...)
- ▶ **first implementation of single-reference CCSD using tensor trains**

acknowledgements

M. G. Kozlov
I. G. Kozhevnikov
T. A. Isaev
D. A. Maltsev
A. N. Petrov
A. V. Stolyarov
V. F. Khrustov

M. Athanasakis-Kaklamanakis
M. Au
A. Borschevsky
V. V. Flambaum
G. Neyens

questions?

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